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ABSTRACT

Exercises for high school students are presented which entail designing computer programs to enable the computer to identify a geometrical figure as a specific type of triangle or quadrilateral, given its 2-dimensional coordinates. Computer programming skills required are not sophisticated, but a solution requires careful analysis of necessary and sufficient conditions for each figure type. If a computer is not available, flow chart solutions are possible. Flow chart samples of possible solutions to a variety of both triangle and quadrilateral identification problems are presented. (Author/STS)

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Number 19

The Illinois Series on

Educational Applications of Computers

Computer Classification of Triangles and Quadrilaterals--

A Challenging Application

J. Richard Dennis

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Computer Classification of Triangles and Quadrilaterals--A Challenging Application

by J. Richard Dennis

Many high schools have access to a computer of some type. Below are two exercises whose solutions are within the capability of most computers available to schools. Both of these exercises have pedagogical features that distinguish them. The mathematics required is relatively simple but comes from several areas--synthetic geometry, analytic geometry, linear algebra. Each of the exercises lends itself to group analysis and planning, with subsequent individual projects. One of them requires study of a class of geometric figures (the non-simple quadrilaterals) not usually treated in high school courses. Computer programming skills required are not sophisticated, but solution requires careful analysis of necessary and sufficient conditions for each figure type. Even if a computer is not available, these exercises present significant experiences in flow-charting solutions.

Here are the exercises:

1. Given the 2-dimensional coordinates of three points, is the figure a triangle, and if so, what type?
2. Given the 2-dimensional coordinates of four points, is the figure a quadrilateral, and if so, what type?

For students to come to appreciate the subtleties of computer solutions to these exercises, a teacher may wish to have students first consider a series of simpler ones such as:

1. Given coordinates of three points, is the figure a right triangle? (Of course, the right angle might be at any one of the vertices.)
2. Given coordinates of four coplanar points, is the figure a parallelogram? (Four coplanar points

determining <u>one</u> pair of parallel, congruent, segments may not, in fact, determine a parallelogram, as Figure 1 illustrates.)

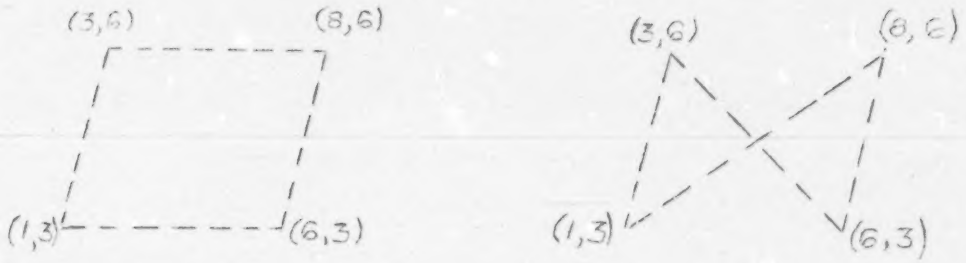


Figure 1

Such questions illustrate the range of alternatives that computer programs must be able to treat in order to accurately identify specific triangles or quadrilaterals.

Once students have created programs to identify particular triangles or quadrilaterals, the task is to develop a strategy for identification of arbitrary ones. This is no trivial task and is worth a major investment of class time. For reference, sample strategies are included below. Strategies for solving such exercises are not necessarily unique. Many working variations are possible.

Triangle Identification

Most high school students are aware of the variety of triangles possible. We are interested in building a network of programs that will identify triangles as scalene, right, acute, obtuse, isosceles, equilateral, and the possible combinations of these. Here is a flow chart for one such network (Figure 2):

TRIANGLES

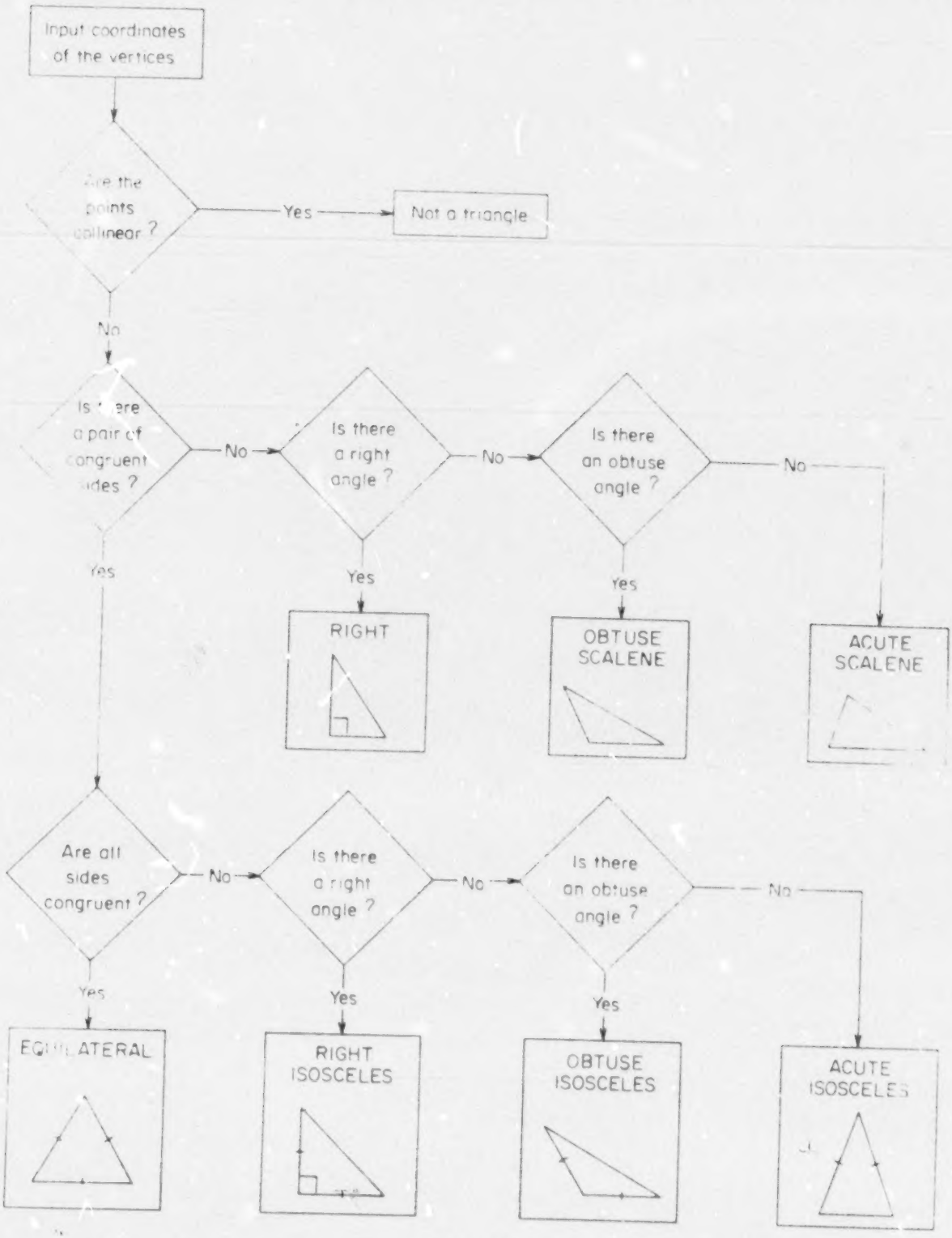


Figure 2

Quadrilateral Identification

High School students are usually very familiar with the varieties of simple convex quadrilaterals, but may be much less familiar with the varieties of simple concave, or non-simple, quadrilaterals and their properties. Not only does identification and classification of such figures present a challenge to their computer skills, it also presents a significant learning experience in geometry.

For this problem, coordinates of the points can be input in the order in which they are connected (first to second, second to third, ..., last to first). If an order for the vertices is not assumed, the quadrilateral is not unique as to type. Initially, however, the program must check to see if there are three adjacent collinear points among the set.

One strategy for identifying a quadrilateral involves an initial categorization into simple convex, simple concave, and nonsimple types. Such a classification is based upon the following properties:

1. The diagonals of plane quadrilateral ABCD are either parallel or they intersect at point P such that:

   P is the image of A under translation $(C-A)x$, for some real number x

   and

   P is the image of B under translation $(D-B)y$, for some real number y

   where, for example, C-A is the translation (vector) that maps A onto C.

2. For the values of x and y determined above, the quadrilateral is:

   a. simple and convex if and only if $0 < x < 1$ and $0 < y < 1$

b. simple and concave if and concave if and only if just one of $0 < x < 1$ or $0 < y < 1$

c. nonsimple if and only if both x and y are not between 0 and 1, or are not defined in which case the diagonals are parallel.

Such a gross classification is shown schematically in Figure 3.

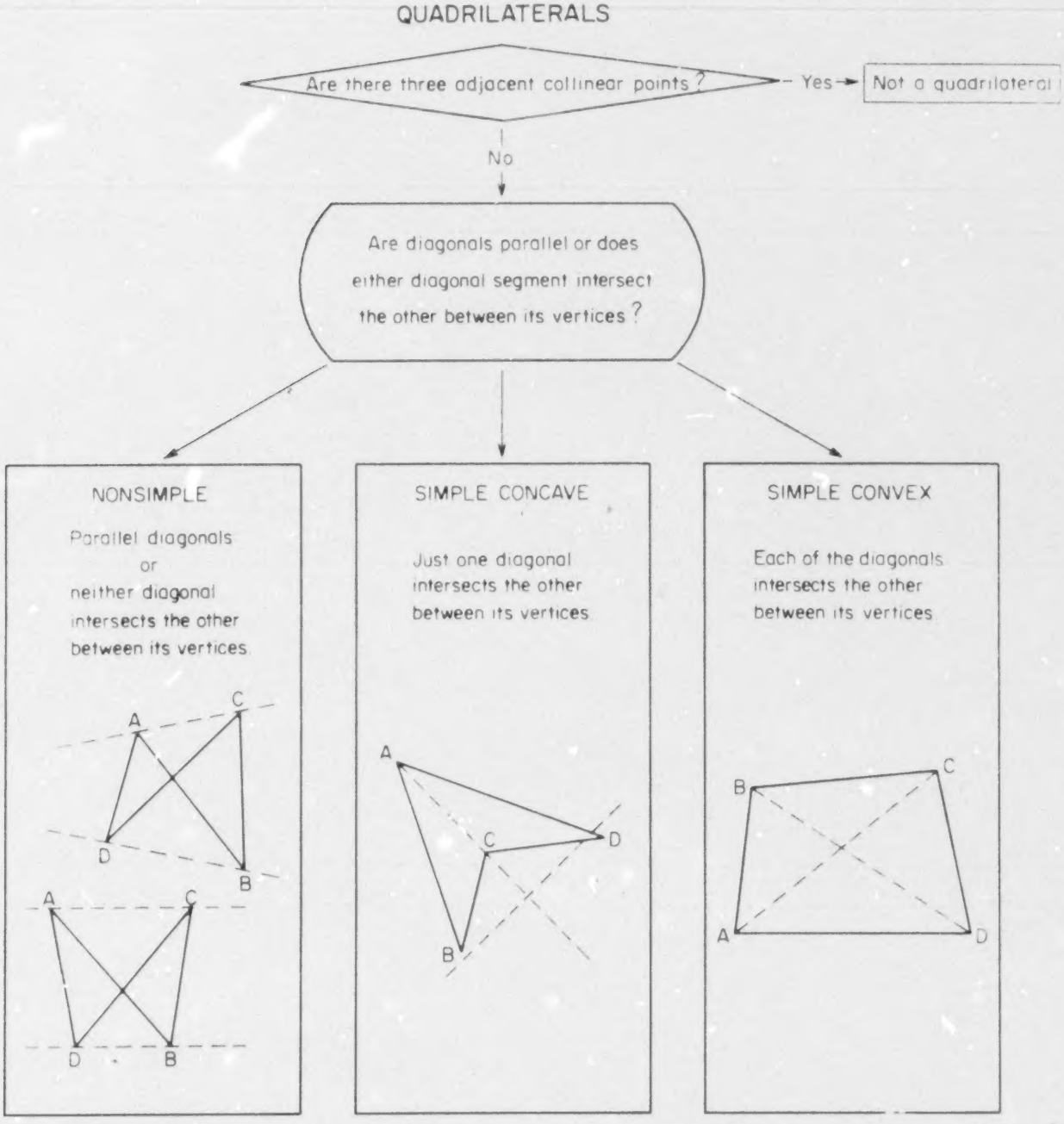

QUADRILATERALS
Are there three adjacent collinear points?
Yes
Not a quadrilateral
No
Are diagonals parallel or does either diagonal segment intersect the other between its vertices?
NONSIMPLE
Parallel diagonals or neither diagonal intersects the other between its vertices
SIMPLE CONCAVE
Just one diagonal intersects the other between its vertices
SIMPLE CONVEX
Each of the diagonals intersects the other between its vertices
A
B
C
D

Figure 3

Once the initial classification has been accomplished, we can turn our attention to the identification of the types of figures in each class. Simple convex quadrilaterals are probably the most familiar. One scheme for distinguishing the simple convex quadrilaterals from among themselves is shown in Figure 4. The initial test in this scheme is to determine the number of pairs of parallel sides.

SIMPLE CONVEX QUADRILATERAL

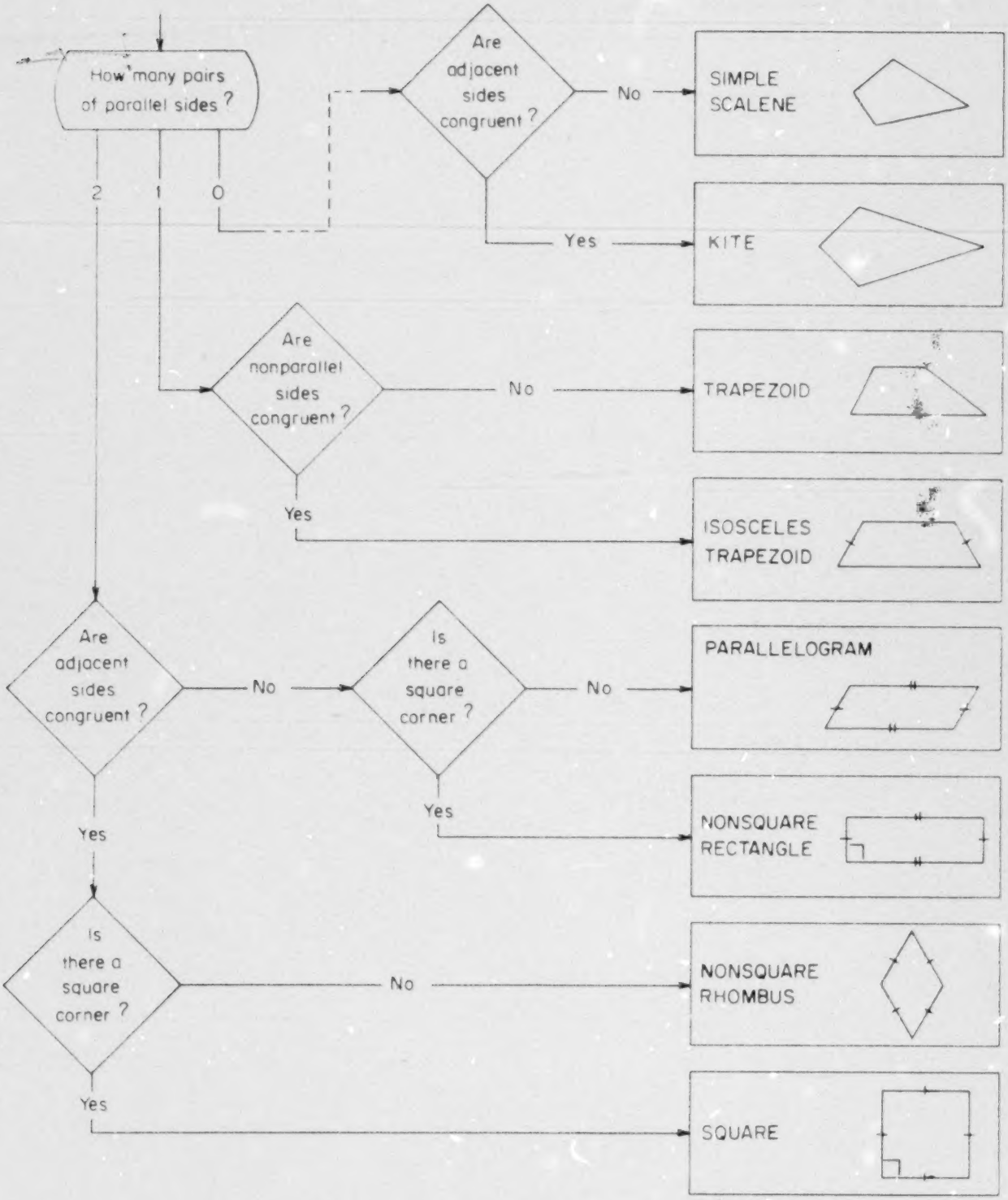


Figure 4

Simple concave quadrilaterals may be less familiar to students but are easy to distinguish among themselves due to the lack of a large number of varieties of such figures. (See Figure 5)

CONCAVE QUADRILATERALS

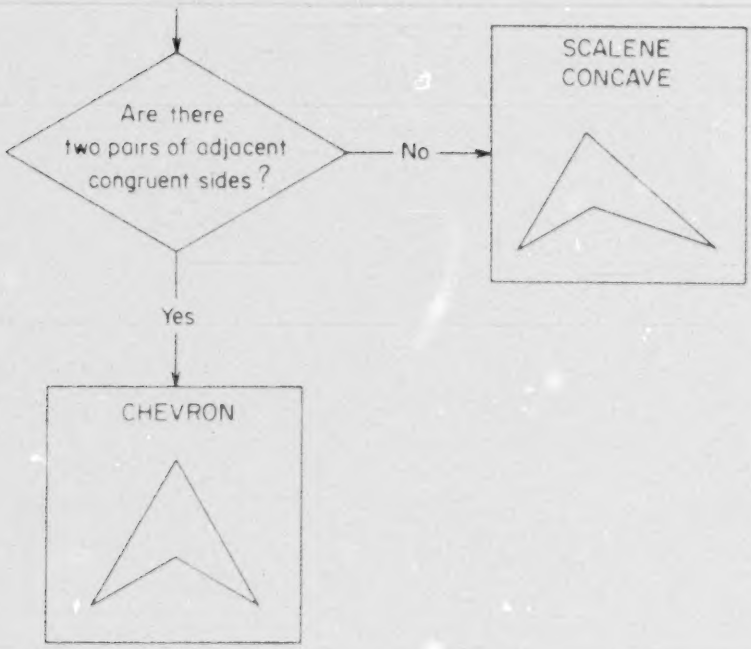


Figure 5

"Bowtie" quadrilaterals usually are unknown to high school students. For this reason, an introductory period of experimental sketching may be in order to acquaint them with the varieties of such figures. One scheme for distinguishing bowties among themselves is initially based on the presence or absence of parallel diagonals, and subsequently, on the presence or absence of a pair of parallel sides. Figure 6 shows the complete scheme for classifying bowtie quadrilaterals.

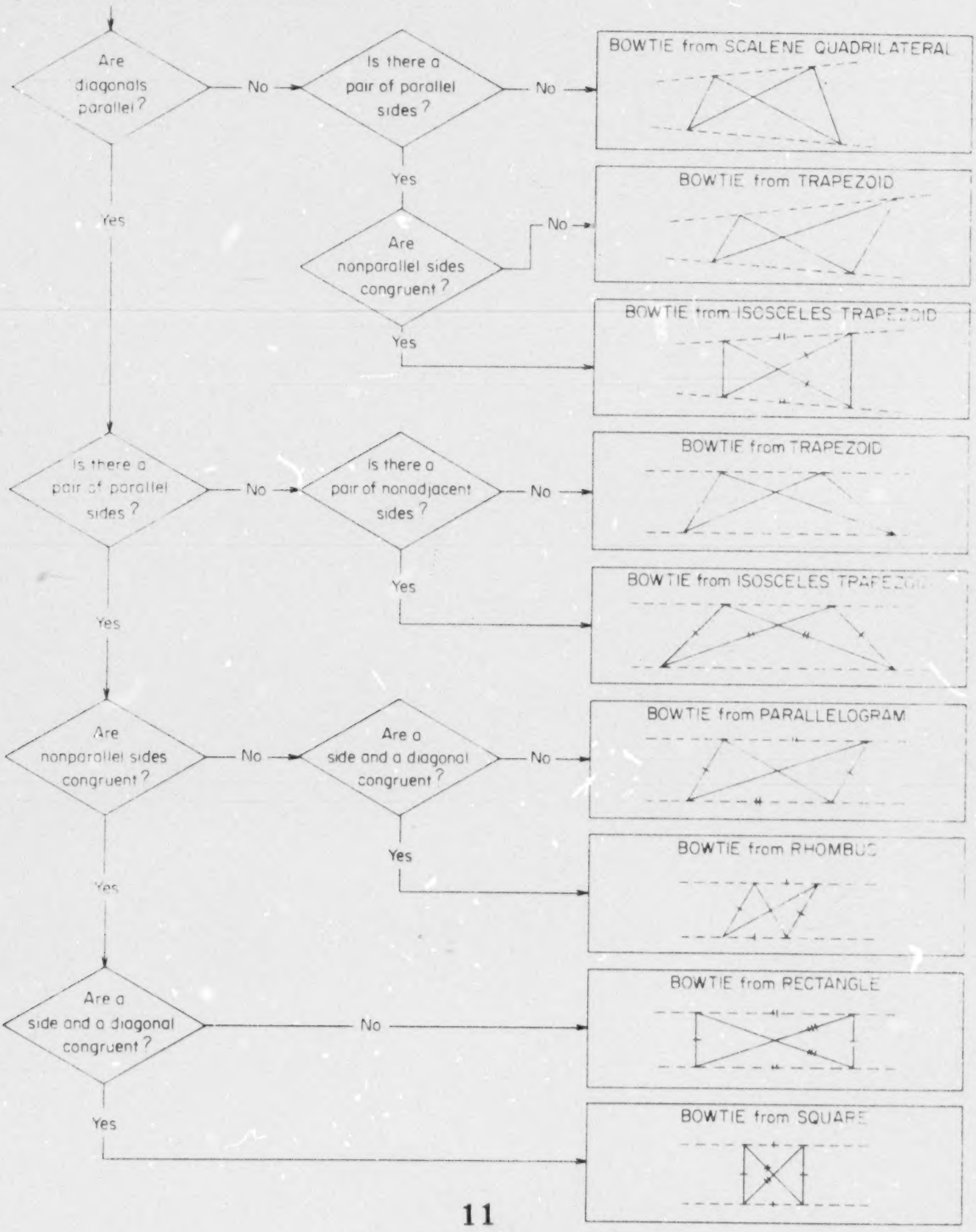

NONSIMPLE QUADRILATERALS
Are diagonals parallel?
No
Is there a pair of parallel sides?
No
BOWTIE from SCALENE QUADRILATERAL
Yes
Yes
Are nonparallel sides congruent?
No
BOWTIE from TRAPEZOID
Yes
BOWTIE from ISOSCELES TRAPEZOID
Is there a pair of parallel sides?
No
Is there a pair of nonadjacent sides?
No
BOWTIE from TRAPEZOID
Yes
BOWTIE from ISOSCELES TRAPEZOID
Yes
Are nonparallel sides congruent?
No
Are a side and a diagonal congruent?
No
BOWTIE from PARALLELOGRAM
Yes
BOWTIE from RHOMBUS
Yes
Are a side and a diagonal congruent?
No
BOWTIE from RECTANGLE
Yes
BOWTIE from SQUARE

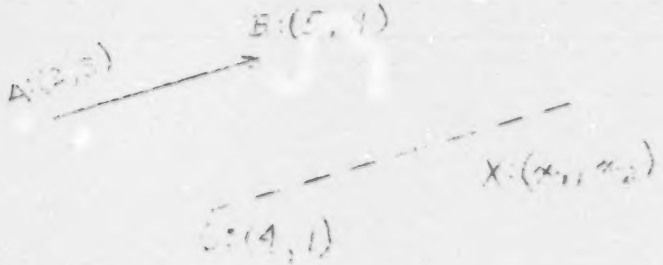
Figure 6

The application of translations made here is relatively easy to foreshadow and is well within the grasp of secondary mathematics students. The basic idea here is that Y - X is <u>that</u> translation (slide) which maps X onto Y.

Now translations in a plane have pairs of coordinates (sometimes called components) just as points do. For point X with coordinates (x_1, x_2) and point Y with coordinates (y_1, y_2), the coordinates of the translation Y - X are $(y_1 - x_1, y_2 - x_2)$. Further, to find the coordinates of the image of a point under a translation, all one does is to add the coordinates of the translation to the respective coordinates of the "starting" point. Here is an example:

Consider points A: (2,3), B: (5,4), and C: (4,1)

Find the coordinates of X: (x_1, x_2) which is the image of C under the translation from A to B (i.e., B - A).



Solution:

For A and B defined as given, the translation B - A has coordinates:

(5 - 2, 4 - 3) = (3, 1)

The coordinates then of X are:

$x_1 = 4 + 3 = 7$

first coordinate of C — first coordinate of B-A — first coordinate of image

$x_2 = 1 + 1 = 2$

first coordinate of C — first coordinate of B-A — first coordinate of image

So, B - A with coordinates (5-2, 4-3) maps C: (4, 1) to X: (7, 2).

The second idea needed is that of a multiple of a translation. For a particular translation, it makes sense to talk of "the translation that maps points, say, 5 times as far but in the same direction." It is natural to use a multiplication notation here.

For a given translation B - A, and a real number x, the translation that maps points x times as far as B - A does is:

$$(B - A)\,x$$

Notice that when $x > 0$, $(B - A)x$ maps points x times as far as (B - A) and in the same direction. When $x < 0$, $(B - A)x$ maps points x times as far as (B - A) but in the opposite direction. And, when $x = 0$, $(B - A)x$ maps each point onto itself (leaves each point fixed). To illustrate this coordinate-wise, let's expand on our original example:

Let A: (2, 3), B: (5, 4), and C: (4, 1).

Find the image Y: (y_1, y_2) of C under $(B - A)7$.

Solution:

As before, B - A has coordinates (5 - 2, 4 - 3). The coordinates of Y are:

$y_1 = 4 + 3 \cdot 7 = 25$

first coordinate of C — first coordinate of B-A — seven times — first coordinate of image

$y_2 = 1 + 1 \cdot 7 = 8$

second coordinate of C — second coordinate of B-A — seven times — second coordinate of image

These basic ideas are the basis of the algebra of translations from a coordinate point of view. To illustrate a quadrilateral diagonal intersection solution, consider the following:

Let A: (2, 3), B: (5, 4), C: (1, 9), D: (6, 7).

Consider quadrilateral ABCD. Do the diagonals intersect? If so, where?

Solution:

The diagonals are $\overline{A\ C}$ and $\overline{B\ D}$.

If the diagonals intersect, the point P: (p_1, p_2) is such that:

P is the image of A under $(C - A)x$, for some x

and

P is the image of B under $(D - B)y$, for some y

We wish to find x and y. Coordinate-wise, we will have:

$p_1 = 2 + (1 - 2)\cdot x$ AND $p_1 = 5 + (6 - 5)\cdot y$

first coordinate of A; first coordinate of (C-A); times x

first coordinate of B; first coordinate of (D-B); times y

$p_2 = 3 + (9 - 3)\cdot x$ AND $p_2 = 4 + (7 - 4)\cdot y$

second coordinate of A; second coordinate of C-A; times x

second coordinate of B; second coordinate of (D-B); times y

Now, for P to be the intersection point both values of p_1 must agree and both values of p_2 must agree. So we have:

$$2 + (1 - 2)x = 5 + (6 - 5)y$$

and

$$3 + (9 - 3)x = 4 + (7 - 4)y$$

This is an old friend--a pair (system) of linear equations in two variables! All we need do is to solve for x and y. When you do this you will find that:

$$x = -\frac{8}{9} \text{ and } y = -\frac{19}{9}$$

So, P is the point (p_1, p_2) such that

$$p_1 = 2 + (1 - 2) \cdot (-\frac{8}{9}) = 2\frac{8}{9}$$

$$p_2 = 3 + (9 - 3) \cdot (-\frac{8}{9}) = -2\frac{1}{3}$$

(Notice that these values agree with the alternate set:)

$$p_1 = 5 + (6 - 5) \quad (-\frac{19}{9}) = 2\frac{8}{9}$$

$$p_2 = 4 + (7 - 4) \quad (-\frac{19}{9}) = -2\frac{1}{3}$$

The important observation for the project discussed here is that neither x nor y is between 0 and 1! So, the quadrilateral ABCD is nonsimple (i.e., a bowtie one whose diagonals intersect "outside" the figure).

By generalizing these few techniques one has all the algebraic machinery needed to carry off the geometry/computer project discussed earlier. This algebra of points and translations can be introduced in only 3 - 4 lessons (with an exercise sheet for each). Some prior graphing of points is desirable. Samples of material presenting transformations, their properties, and applications in the coordinate plane may be found in Chapters 7, 9, and 11 of Geometry--A Modern Course, by Beberman, Dennis, Wolfe, and Zwoyer. (D.C. Heath & Co., Lexington, Mass., 1971.)

It is rather difficult to find somewhat interesting, but challenging activities that involve geometry concepts and computers. The material presented here may be explored from several points of view, and to a variety of levels of intensity. A teacher wishing to do CAI lessons in geometry in which the student would be asked to draw geometric figures (either on graph paper or on a graphics capable terminal), and the computer would be expected to identify the figure in order to provide instructive feedback to the student, would use precisely the procedures presented here as a basis for the required answer checking algorithms.* One set of such lessons is available on the PLATO system based at the University of Illinois Champaign-Urbana Campus (request lesson GEOMETRY).

*J. Richard Dennis, "Geometry Via PLATO," The Instructor, November, 1968, pp. 116.
_____, Teaching Selected Geometry Topics via a Computer System, unpublished doctoral thesis, University of Illinois, June, 1968.